

Topology in QCD with 4 flavours of dynamical fermions ^{*}

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We study the topological properties of full QCD with four flavours of dynamical staggered fermions. In particular the topological susceptibility is measured and the problem of the determination of its first derivative is discussed.

1. INTRODUCTION

Topology plays an essential role in determining the low energy features of strong interaction physics. In this paper we present a lattice study of the topological properties of QCD in presence of four flavours of degenerate dynamical staggered fermions.

A relevant quantity is the topological susceptibility, defined in the continuum as

$$\chi \equiv \int d^4x \partial_\mu \langle 0 | T \{ K_\mu(x) Q(0) \} | 0 \rangle, \quad (1)$$

where $K_\mu(x)$ is the Chern current and $Q(x) = \partial_\mu K_\mu(x)$ is the topological charge density,

$$Q(x) = \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a(x) F_{\rho\sigma}^a(x). \quad (2)$$

Eq. (1) defines the prescription for the singularity of the time ordered product when $x \rightarrow 0$ [1].

The value of χ in the quenched theory is related to the η' mass by the Witten-Veneziano formula. It has been successfully measured on the lattice [2], confirming the Witten-Veneziano prediction.

In full QCD with spontaneous chiral symmetry breaking, χ is related to the quark condensate [3]

$$\chi = \frac{m_q}{N_f} \langle \bar{\psi}\psi \rangle_{m_q=0} + o(m_q), \quad (3)$$

where m_q is the quark mass and N_f the number of flavours. We want to test Eq. (3) for $N_f = 4$ and different values of m_q .

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Another relevant quantity is the slope at $q^2 = 0$ of the topological susceptibility, χ' , defined as

$$\chi' \equiv \frac{d\chi(q^2)}{dq^2} \Big|_{q^2=0} = \frac{1}{8} \int d^4x \langle Q(x) Q(0) \rangle x^2, \quad (4)$$

where $\chi(q^2) = \int d^4x e^{iq \cdot x} \langle Q(x) Q(0) \rangle$. In the quenched theory the consistency of the Witten-Veneziano mechanism requires χ' to be small. Instead in full QCD it is expected to be larger and related to the singlet axial charge (and so to the proton spin crisis) [4]. Therefore its determination is of particular importance. Unfortunately, techniques which work well for the lattice determination of χ , are not straightforwardly applicable to the measurement of χ' . We will present a preliminary estimate of χ' and discuss perspectives for future refinements.

2. TOPOLOGICAL SUSCEPTIBILITY

We have simulated the full theory using the HMC algorithm with 4 flavours of staggered fermions at $\beta = 5.35$ and four different values of the bare quark mass, $a \cdot m_q = 0.010, 0.015, 0.020, 0.050$, performing for each mass value respectively 6000, 1500, 1000 and 3000 units of molecular dynamics time. We have used a $16^3 \times 24$ lattice and the Wilson action for the pure gauge sector.

The field theoretical method has been used to determine χ . Given a discretization $Q_L(x)$ of the topological charge density, we define the lattice susceptibility:

$$\chi_L = \sum_x \langle Q_L(x) Q_L(0) \rangle = \frac{\langle Q_L^2 \rangle}{V}. \quad (5)$$

Q_L is related by a finite multiplicative renormaliza-

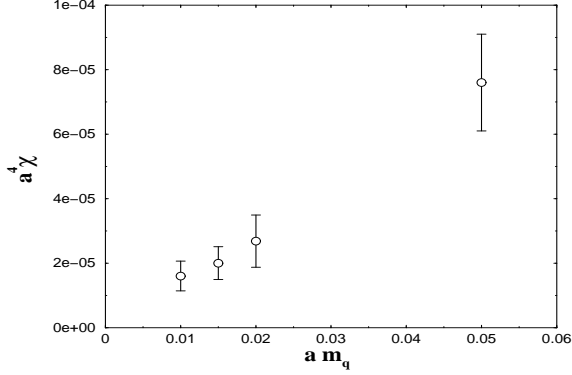


Figure 1. Topological susceptibility in lattice spacing units at different bare quark masses and $\beta = 5.35$.

tion Z to the continuum topological charge³. Moreover χ_L in general does not meet the continuum prescription used in Eq. (1) and, due to contact terms in the product $Q(x)Q(0)$ at small x , a further additive renormalization appears

$$\chi_L = a^4 Z^2(\beta) \chi + M(\beta). \quad (6)$$

The heating method [6] has been used to evaluate the renormalizations. Z is determined by measuring $\langle Q_L \rangle$ on a sample of thermalized configurations obtained by heating a semiclassical one-instanton configuration. Similarly M is obtained by measuring χ_L on a sample with zero topological background, using the fact that Eq. (1) leads to $\chi = 0$ in the trivial topological sector. Smearing techniques have been used to improve the operator Q_L [7], in order to reduce the renormalizations and obtain a better accuracy on χ after subtraction. We have performed 2 smearing steps, starting from a standard discretization of Q , with a smearing coefficient $c = 0.9$, as reported in [2].

In Fig. 1 we report the results obtained for $a^4 \chi$ versus the quark mass. The clear dependence of $a^4 \chi$ on m_q disappears when going to physical units. We have fixed the scale by measuring the string tension, obtaining $a^2 \sigma = 0.073(7)$ at $a \cdot m_q = 0.05$ and $a^2 \sigma = 0.033(4)$ at $a \cdot m_q = 0.01$. From these values, assuming $\sqrt{\sigma} = 440$ MeV, we obtain $a(a \cdot m_q = 0.01) = 0.081(5)$ fm and $a(a \cdot m_q = 0.05) = 0.0121(6)$ fm, from which $(\chi)^{1/4}(a \cdot m_q = 0.01) = (153 \pm 16)$ MeV and $(\chi)^{1/4}(a \cdot m_q = 0.05) = (153 \pm 11)$ MeV.

³In the full theory mixings of Q_L with fermionic operators appears, which anyway can be shown to be negligible [5].

We cannot rule out possible systematic errors coming both from the poor sampling of topological modes at the lowest quark masses [8] and from the determination of the physical scale. Indeed at $a \cdot m_q = 0.01$ we have determined the lattice spacing also by measuring m_ρ and m_π , obtaining a different result, $a = 0.101(5)$ fm. Using this value we obtain $(\chi)^{1/4}(a \cdot m_q = 0.01) = (123 \pm 10)$ MeV, which is closer to the theoretical expectation coming from Eq. (4), $(\chi)^{1/4} \simeq 110$ MeV.

3. ESTIMATE OF χ'

On the lattice we can define

$$\chi'_L = \frac{1}{8} \sum_x \langle Q_L(x) Q_L(0) \rangle x^2. \quad (7)$$

A relation similar to Eq. (6) holds in this case [9]:

$$\chi'_L = a^4 Z^2(\beta) \chi' + M'(\beta). \quad (8)$$

While Z in the previous equation is the same appearing in Eq. (6), since it is purely related to the renormalization of the topological charge, M' is a new additive renormalization, containing mixings of χ'_L to operators of equal or lower dimension. More generally, defining the two-point correlation function of the topological charge density on the lattice, $\langle Q_L(x) Q_L(0) \rangle$, one can write

$$\langle Q_L(x) Q_L(0) \rangle = a^8 Z^2 \langle Q(x) Q(0) \rangle + m(x); \quad (9)$$

$m(x)$ indicates mixings with terms of equal or lower dimension in the Wilson OPE of $\langle Q_L(x) Q_L(0) \rangle$, and is related to M and M' in the following way:

$$M = \sum_x m(x); \quad M' = \sum_x m(x) x^2. \quad (10)$$

M' cannot be determined by the heating method, since in this case the continuum χ' is not constrained to be zero in the trivial topological sector. Therefore the techniques used for χ cannot be straightforwardly applied to the determination of χ' . In principle M' can also be computed by lattice perturbation theory, but in practice this approach is not particularly successful, especially when dealing with smeared operators for which the convergence properties of the perturbation series worsen. We stress that also cooling techniques, which can usually be used to determine χ , fail in the determination of χ' : only the zero-moment

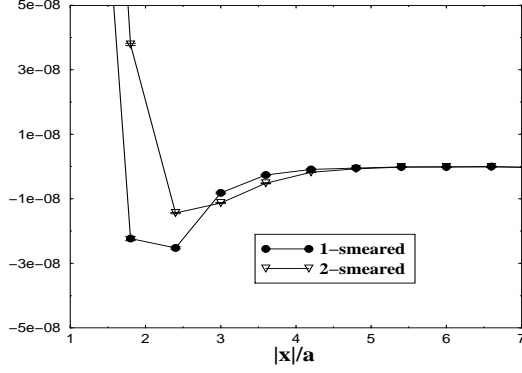


Figure 2. $\langle Q_L(x)Q_L(0) \rangle$ for the operators after one and two smearing steps, at $a \cdot m_q = 0.01$

of the two-point function $\langle Q(x)Q(0) \rangle$, i.e. χ , is topologically protected, since it can be expressed in terms of the global topological charge, $\chi = \langle Q^2 \rangle / V$, and Q is quasi-stable under cooling. This is not the case for higher moments, and in particular for χ' .

One possibility is to use an improved topological charge operator for which M' , even if not computable, is known to be small and thus negligible in Eq. (8). If we follow this ansatz for the 2-smeared operator used previously we obtain, from our simulation at $a \cdot m_q = 0.01$, $\sqrt{|\chi'|} \sim 20$ MeV, in good agreement with the value expected from sum rules [4], $\sqrt{|\chi'|} = (25 \pm 3)$ MeV. However, the systematic error deriving from neglecting M' , M'/χ' , can be estimated to be of the same order of magnitude as $M/\chi \simeq 40\%$ at $a \cdot m_q = 0.01$, that is still quite large.

A better estimate of χ' requires deeper knowledge of the two-point correlation function. In the continuum $\langle Q(x)Q(0) \rangle$ is known to be negative, by reflection positivity, for $x > 0$, and positive and singular for $x = 0$. Similarly, when using a lattice action which preserves reflection positivity, we expect $\langle Q_L(x)Q_L(0) \rangle < 0$ whenever the two operators $Q_L(0)$ and $Q_L(x)$ do not overlap. This is clear in Fig. 2, where a determination at $a \cdot m_q = 0.01$ of $\langle Q_L(x)Q_L(0) \rangle$ averaged over spherical shells of width $\delta x = 0.6a$ is reported for 1 and 2 smearings.

The information on $\langle Q_L(x)Q_L(0) \rangle$ is not enough to extract $\langle Q(x)Q(0) \rangle$: according to Eq. (9), also $m(x)$ is needed. However we know that $m(x)$ comes from contact terms, so it must be zero at large $|x|$. Therefore, for a given operator $Q_L(x)$, there must be an x_0 such that, for $|x| > x_0$, $m(x)$ can be ignored in

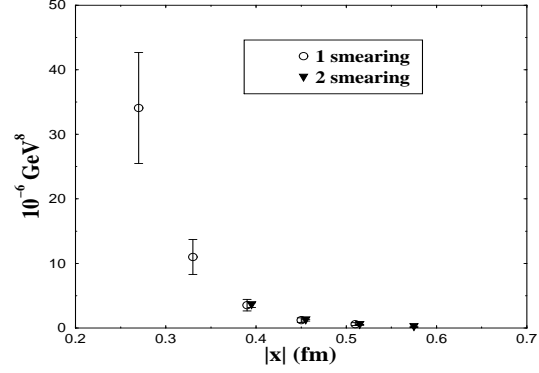


Figure 3. $-\langle Q(x)Q(0) \rangle$ from 1 and 2-smeared operators at $a \cdot m_q = 0.01$

Eq. (9) and $\langle Q(x)Q(0) \rangle$ can be easily extracted, the value of Z being known from the determination of χ .

A practical way to extract x_0 is the following: $\langle Q_L(x)Q_L(0) \rangle$ is determined for two operators Q_{L1} and Q_{L2} and one looks for a plateau at large $|x|$ in the ratio of the two functions, corresponding to the squared ratio of the multiplicative renormalizations, $(Z_1/Z_2)^2$. In this way we have determined $\langle Q(x)Q(0) \rangle$ for 1 and 2 smearing steps at large $|x|$ and $a \cdot m_q = 0.01$, as shown in Fig. 3. There is good agreement between the two determinations, as expected. Work is in progress to extrapolate the information at large $|x|$ to smaller distances, thus allowing a more careful determination of χ' .

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